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LETTERS TO THE EDITOR

COMMENTS ON "VIBRATION FREQUENCIES OF SIMPLY SUPPORTED POLYGONAL SANDWICH PLATES VIA KIRCHHOFF SOLUTIONS"

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The writers wish to congratulate Dr Wang for his extremely interesting paper [1]. The fact that exact vibration solutions for sandwich plates with simply supported, straight edges can be obtained by using exact Kirchhoff plate vibration solutions and ultimately from the classical vibrating membrane problem is a remarkable one.

The analogy is also valid in the case of doubly connected plates, as long as both boundaries are constituted of rectilinear simply supported sides.

Another fundamental property is also of interest. It has been pointed out recently [2] that the fundamental frequency coefficient λ_{11} of a membrane of arbitrary shape and that of a membrane of circular shape, α_0, are related by the inequality [3]

$$\lambda_{11} < \alpha_0/a_0, \tag{1}$$

where $\alpha_0 = 2{\cdot}4048$ (the first root of the Bessel function of the first kind and of order zero) and a_0 is the coefficient of the first term of the infinite series which maps a unit circle on to the arbitrary shape. Similarly, in the case of a doubly connected membrane of fixed edges,

$$\lambda_{11} < \alpha_{11}/a_0, \tag{2}$$

where α_{11} is the frequency coefficient of the corresponding circular, annular membrane and a_0 is the coefficient of the ξ term in the Laurent expansion, which maps the given doubly connected membrane on to a circular annulus in the ξ-plane.

In view of the important theorem proved by Wang [1], it turns out that the fundamental frequency coefficients of simply supported, sandwich plates of rectilinear sides are related to the fundamental frequency coefficient of a circular membrane by the inequality (1) in the case of a simply connected domain, and by the inequality (2) if the plate is doubly connected with both edges simply supported and made of rectilinear sides.

In the case of a simply supported plate (sandwich or Kirchhoff) with re-entrant corners, the membrane analogy should be considered as a first order mathematical model, since a stress concentration field is generated at a re-entrant corner. This, in turn alters the value of the natural frequencies obtained from vibrating membrane model.

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AUTHOR'S REPLY

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The author would like to thank Professor Laura and Ms Rodríguez for their appreciation of the author's work and for their remarks on the existence of some inequalities involving the fundamental frequency of vibrating membranes that are applicable to simply supported, polygonal plates.

Furthering this discussion, the author would like to point out the work done by Pnueli [1]. Pnueli considered the eigenvalue problem governed by the biharmonic equation

$$\nabla^2\nabla^2 w - \lambda^2 w = 0, \qquad \text{in the region } \Omega, \tag{1}$$

with the boundary conditions

$$w = 0, \qquad \text{on } \Gamma, \text{ the boundaries of } \Omega, \tag{2}$$

and

$$\partial w/\partial n = 0, \qquad \text{on } \Gamma_c, \tag{3}$$

(Γ_c denotes the portion of the boundary of Ω over which the normal derivative vanishes), and

$$\nabla^2 w = 0, \qquad \text{on } \Gamma_s, \tag{4}$$

where $\Gamma_s \cup \Gamma_c = \Gamma$. Note that the foregoing eigenvalue problem covers the vibration problem of plates the edges of which are either simply supported, or clamped, or of mixed combinations of simply supported and clamped edges.

Here the author cites the three theorems that Pnueli had stated on the inequalities associated with the above eigenvalue problem.

Theorem 1. A lower bound to the lowest eigenvalue of equation (1) with the boundary condition of equations (2) and (3), over the two-dimensional region bounded by Γ, is that obtained from the solution of the same equations over a circular two-dimensional region of the same area.

Theorem 2. A lower bound to the lowest eigenvalue of equation (1) with the boundary conditions of equations (2) and (4), over the two-dimensional region bounded by Γ, is that obtained from the solution of the same equations over a circular two-dimensional region of the same area.

Theorem 3. A lower bound to the Nth eigenvalue of equation (1) with the boundary condition of equations (2) and (4), over the two-dimensional region bounded by Γ, is that obtained from the solution of the equation (1) with the boundary conditions of equations (2) and (4) over a circular region of $1/N$ the area of the original region.

It is clear that the lower bounds given by Theorems 2 and 3 are directly applicable to the class of plates that the author has considered; i.e., general polygonal plates with simply supported edges. The author hopes that the above theorems will be useful to the writers.

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